

A Mathematical Model of the Fall of Man and the Atonement of Christ

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Introduction

Most humans believe in the reality of a spiritual world, but little has been done to attempt to quantify the essential elements of this realm, if it indeed can be done. James Clerk Maxwell made the following statement:

It was a great step in science when men became convinced that, in order to understand the nature of things, they must begin by asking, not whether a thing is good or bad, noxious or beneficial, but of what kind it is? And how much is there of it? Quality and Quantity were then first recognized as the primary features to be observed in scientific inquiry.¹

Although obvious difficulties immediately arise when attempting to apply quantitative concepts to the spiritual world, it may be that in metaphysics, as in physics, progress in understanding can be achieved when

¹ J. C. Maxwell, "Address to the Mathematical and Physical Sections of the British Association, Liverpool, 15 Sept 1870," in *The Scientific Papers of James Clerk Maxwell*, ed. W. D. Niven (1890; reprint, New York, Dover Publications, 2003), 2:217.

such is attempted. This has certainly held true in the physical universe, as witnessed by the remarkable predictive ability of the mathematical sciences. While the mathematical model presented in this paper is not intended to be taken seriously in any rigorous way, the intention is that it may shed some light on the spiritual dynamics entailed in the Christian faith, and perhaps stimulate further work by others.

Quantification of Moral Perfection

In Genesis 2:17 the first man and woman are warned that they must not eat of the tree of the knowledge of good and evil, “for when you eat from it you will certainly die.”² What is death, in the sense implied in the Bible? It is apparent that the meaning of death is not as simple as physical death, although physical death seems to be related to it. It is written that “sin, when it is full-grown, gives birth to death” (James 1:15), but that those who trust in Christ “have been brought from death to life” (Romans 6:11). Since this can clearly happen before physical death of an individual occurs, it is obviously something more than physical death that is being described here. Because this is clearly a spiritual concept, I will refer to this kind of death as “spiritual death.” Furthermore, this death is evidently directly related to the act of committing sin, i.e., turning away from God and interest in the welfare of others, toward interest in the self at the expense of others.

In this work, sin is considered to be synonymous with death. Death in this sense, however, can occur by degrees, in the same manner that a tree may be only “partly dead”—it may have mostly dead branches, but some part that is still living. Conversely, when sin is removed in some way, there is an increase in life: “Awake you that sleep, and arise from the dead, and Christ shall give you light” (Ephesians 5:14).

In another sense, sin is like a weight, so that when a being is “totally weighed down,” it is dead. If the spiritual realm is to be understood in a concrete, rational way, it seems to me that sin must be a quantity that has substance to it. Mackintosh quotes Anselm in describing the “weight of sin”:

The elusive greatness of the Divine sacrifice is the measure of the danger that threatened once but threatens no longer. There is in forgiven [people] a shuddering thankfulness, as they look back and draw breath in the peace of reconciliation, which seals the horror of the darkness in which we should have sunk but for the dearly paid

² Unless specified otherwise, all quotations are from the New International Version of *The Bible*.

mercy of God. *Nondum considerasti quanti ponderis sit peccatum*, said Anselm in his dialogue [“you have not yet considered how great the weight of sin is”]; and the words have repeated themselves ever since, judging facile theories.³

Thus, the “weight of sin” is spoken of as though sin has substance. Anyone who has had a “burden of guilt” knows that it can feel like a literal weight on one’s shoulders, but it is felt in the spirit. Suppose that it can be thought of as a substance, but the substance is spiritual.⁴ If so, then the increase or decrease of the “weight” or substance of sin on an individual would be simply the rate of at which he or she sins, less the rate at which the sin is taken away though the consequences of repentance.

In Judeo-Christian theology, it is clear that sin is important to God. This is apparently not because it simply *bothers* Him, but that it hurts not only others, but it hurts *the offender*, and He doesn’t want anyone to be hurt. So we are told to “be holy, as I am holy” (Leviticus 11:44), because otherwise there are inevitable consequences—the consequences are spiritual death for “the wages of sin is death” (Romans 6:23).

People often have difficulty comprehending the Christian concepts of the Fall of Man and the Atonement of Christ. One misconception and gross simplification is that “Man sinned, which made God angry, but Jesus agreed to step in and take the heat, thus appeasing God.” I believe we are dealing with something much more profound than that—a mysterious and potent dynamic that takes place in the realm of the spirit. C. S. Lewis was addressing this dynamic when he made the following statement:

Only a bad person needs to repent. Only a good person can repent perfectly. The worse you are the more you need it and the less you can do it. The only person who could do it perfectly would be a perfect person, and he would not need it.⁵

I have heard others mention that once they fall in a certain way and do something immoral and wrong, it is easier to repeat the action again, even though they feel it is wrong. I have noticed this with myself. It becomes harder to repent once I have “fallen” morally in some way. Here we need to make a distinction between “feeling bad” or “sorry” for having done

³ H. R. Mackintosh, *The Christian Experience of Forgiveness* (London: Nisbet, 1927), 159.

⁴ It is the author’s opinion that spiritual substance is closely related to, or possibly equivalent to, the *res cogitans* or “thinking things” proposed by Renee Descartes.

⁵ C. S. Lewis, *Mere Christianity* (C.S. Lewis Pte, Ltd., 1952), Book II, Chapter 4.

something wrong, as opposed to truly repenting, which means to stop doing that thing and “change direction” by changing one’s behavior.

What sense does it make to punish someone for the offense of another? It makes no sense at all. It does make sense, however, if the offense is thought of as a debt of some kind, for it is certainly understandable that one person could pay another’s debt. It may be that the idea of a debt is indeed the best analogy. However, note that a debt can be thought of as a kind of “substance” that is independent of the existence of the being from whom it is owed. If a person dies, for example, their debts have, historically, often been passed on to their descendants. Debt is a substance only in the sense that debt is an independent quantity, and that it can be paid for by another.

We can easily feel bad about having done something that we believe to be morally wrong—that requires only a conscience, which is possessed by nearly all humans (except perhaps those uncommon individuals who have some deficiency—in the same way, for example, that we might talk about someone having a defective heart).⁶ But feeling bad and sorry do not equate to “not doing it again.” It is also worth noting here that the sins of thought in humans are so frequent that we often do not recognize them as such, and usually think better of ourselves than we ought.

The statement by Lewis, quoted above, led the author to a pair of differential equations that could, in the imperfect and oversimplified manner of all models, perhaps explain the essential dynamics of sin and illustrate why a species with free will can so easily arrive at a seemingly hopeless predicament.

One last statement should perhaps be made before presenting the mathematical model that forms the substance of this paper: modern physics reveals the inner workings of the physical universe to be remarkably complex and profound. Perhaps even more remarkably, history has shown that it can often be spectacularly well described through mathematical reasoning. What follows is very speculative. However, if what we call the *spiritual* world is indeed a reality, it can be expected to be⁷ at least as difficult as modern physics and perhaps subject to mathematical laws as well.

A Simple Mathematical Model

In what follows, rather than quantifying corruption directly, I will use its complement, which I will call holiness, or perfection. The perfectly holy being is defined to have a perfection of unity, and a being that has no

⁶ It is also possible to “sear” one’s conscience by repeatedly disregarding the advice that one’s conscience gives, but that is yet another matter (e.g. 1 Timothy 4:2).

⁷ C. S. Lewis, *Mere Christianity* (C.S. Lewis Pte. Ltd., 1952), 121.

holiness remaining at all, or a “completely corrupt” being, has a holiness of zero.

For any being, the rate of change of its holiness is the difference between the rate of its increase, which is proportional to the rate of repentance R , minus the rate at which it commits sin, S . Thus, the rate of change of the holiness of man, P_m , is modeled as:

$$\frac{dP_m}{dt} = R_m - S_m \tag{1}$$

Now, if Lewis is right, the rate at which a being can repent depends on its present degree of holiness, P . Again, Lewis claimed “only a perfect being can repent perfectly.” Similarly, it also seems reasonable to assume that a “completely base” or “completely corrupt” being cannot repent at all.⁸ The form of $R(P)$ is of course unknown, but a wide range of possible forms that are consistent with the above two criteria can be described by the following parameterization:

$$R = AP^n \tag{2}$$

where A and n are constants. Similarly, the rate at which a being sins would also depend on its holiness. A reasonable functional form is given by:

$$S = B(1 - P^n) \tag{3}$$

where B is a constant. Some readers might object to the seemingly arbitrary choice of the functional forms given by equations 2 and 3. However, it is important to note that the precise mathematical forms do not matter much; any functions that have the proposed attributes for R and S will give results of the same essential character. If, for the sake of simplicity, we assume that the rate at which a perfectly holy being can repent is equal to the rate at which a completely base being sins, then $A=B$. Furthermore, if we define a convenient nondimensional time

$$t' = \frac{t}{\tau} \tag{4}$$

where τ (the Greek letter tau) is defined (for subsequent convenience of the resulting equation) as the time required for a sinless (initially perfectly holy) being to fall to zero holiness while sinning at the maximal rate $S(P=0)$, then

⁸ One variable that is suppressed here is volition. It is tacitly assumed that the being in question is *trying* to repent. Conscious beings with free will obviously cannot be modeled entirely with simple equations.

all unknown constants vanish except n , and equation 1 takes on the clean form:

$$\frac{dP_m}{dt} = P_m^n - (1 - P_m^n) = 2P_m^n - 1 \tag{5}$$

Figures 1a and 1b show the proposed functions $R(P)$ and $S(P)$ for various values of n .

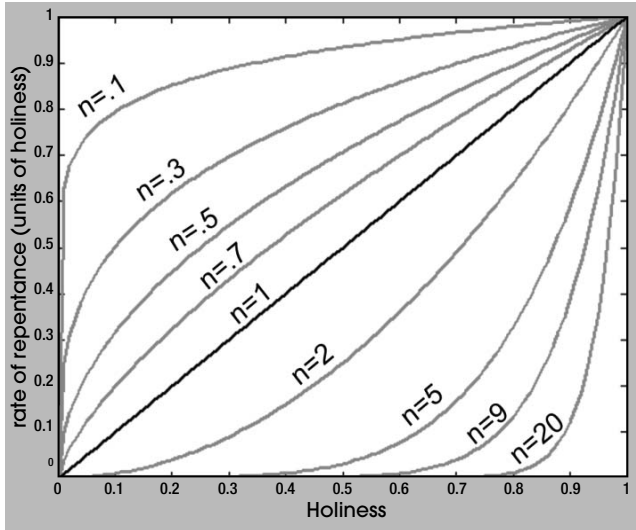


Figure 1a: Rate of repentance R as function of holiness.

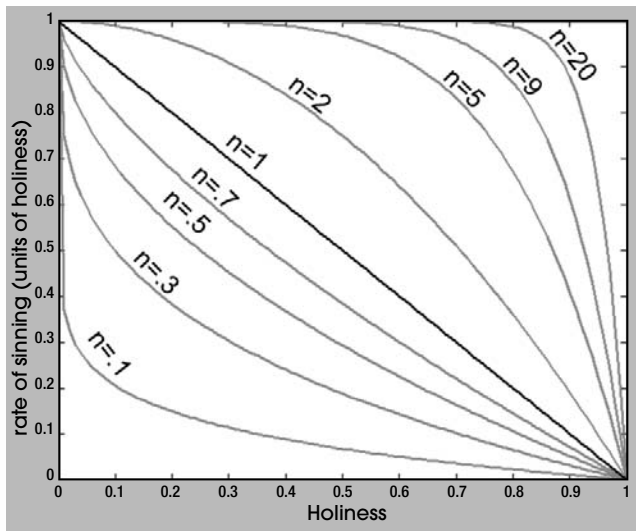


Figure 1b: Rate of sinning S as function of holiness P . Both functions are shown for various values of n .

Note that equation 5 will apply only after man’s holiness has been compromised by sin. Before any sin occurs,

$$\frac{dP_m}{dt'} = 0 \tag{6}$$

This is because the rate of sinning is still zero, and repentance is not needed if there has been no sin. Now we can begin to see the consequences of sin, for various values of n . If $n=1$, equation 5 can be easily integrated to give

$$P_m(t') = \frac{1}{2} [1 + (2P_{m0} - 1) \exp(2t')] \tag{7}$$

where P_{m0} is the initial condition—i.e., the holiness of man at time $t' = 0$. Figure 2 shows the solution given by equation 7 for various initial conditions. For $n=1$, it is clear that man can recover to perfect holiness, in a finite time, by repentance, as long as his initial holiness is greater than $\frac{1}{2}$. If we define $t'=0$ to be the time just after the first sin is committed, then the model predicts that man can recover his holiness as long as the original sin did not cause a reduction in holiness by more than a factor of two.

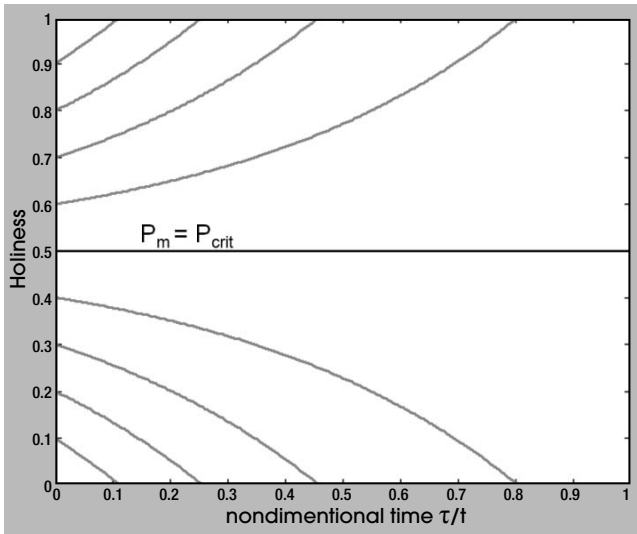


Figure 2: $P(t')$ for $n=1$ and various initial conditions. See text.

Now, if sin is *quantifiable* it is also *quantized*, because it occurs in individual events. It is impossible to estimate the magnitude of the loss of holiness due to an individual sin. On the one hand, some Scriptures indicate that certain sins are more severe than others (cf. Hebrews 10:29), while James states that if anyone is guilty of one sin he is guilty of them all (James 2:10).

Let us make the plausible assumption that the original sin makes only a small reduction in holiness. Then, if n is small, so that $R(P)$ and $S(P)$ are weak functions of P near $P=1$, man can recover his holiness by himself.⁹ But if n is large, R and S are strong functions of P near $P=1$, so that the ability to repent is greatly diminished, and the rate of sinning increases greatly, if P becomes smaller than unity as the result of a single, small loss of holiness due to some original sin (see for example, the case of $n=20$ in figures 1a and 1b). In fact, equation 5 indicates that there is a critical level of holiness below which Man cannot by himself recover from his Fall, given by

$$P_{crit} = \left(\frac{1}{2}\right)^{\frac{1}{n}} \quad (9)$$

(see figure 3). In other words, as n increases, the black horizontal trajectory of figure 2 moves upward, closer to $P_m = 1$. If Man's holiness drops below this level, he will become completely corrupt ($P_m = 0$) in a finite time. If n is large, it is possible that the first sin, even though possibly small, will lead to an inevitable fall from holiness in a finite time. We do not know the best

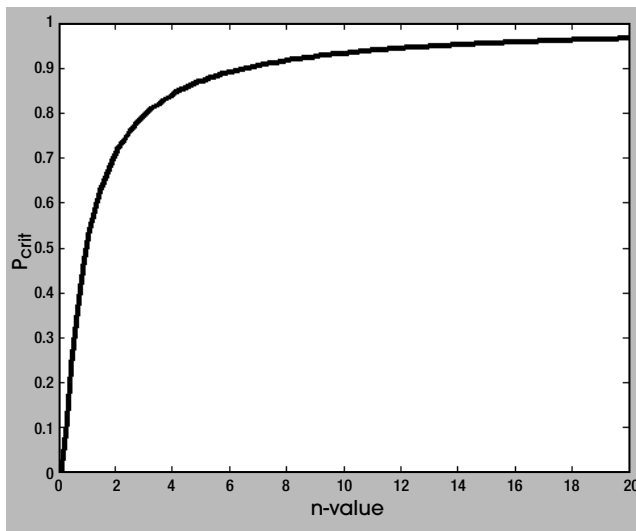


Figure 3: Critical value of holiness below which man cannot recover holiness after original sin, plotted versus the strength n of the state dependence of the rates of sin and repentance.

⁹ Because it is God who forgives sins (e.g. Mark 2:6,7; Luke 5:21), it would still be, in an important sense, *not* by himself, but what I mean is that there need not be someone else taking Man's burden of sin upon Himself.

value for n in regards to humans. However, Scripture states that God told Adam and Eve that if they ate of the fruit of the tree of the knowledge of good and evil, they would surely die (Genesis 2:17).

This suggests that R and S are strong functions of P , near $P=1$, because in that case, even if the magnitude of the first sin is small, the result will be spiritual death—i.e., a complete loss of holiness in a finite time. The solutions to equation (7) for $n>1$ can be written in terms of hypergeometric functions, but it is not necessary to quote those solutions here. The solutions resemble those shown in figure 2, but with faster rate of evolution toward $P=0$ and $P=1$, below and above the critical initial value given by equation (8).

It is reasonable to assume that it is impossible to sin without knowledge of what sin means—in other words, without a knowledge of good and evil, i.e., a comprehension of what it means to serve one's self at the expense of the welfare of others (as James 3:17 says, “Whoever knows what is right to do and fails to do it, *for him it is sin*”). Such knowledge requires a highly developed brain. The fruit of the Tree of Knowledge of Good and Evil is historically represented as an apple, probably because the Latin word *malus*, when used as an adjective, means *evil*, but when used as a noun, means *apple*. Eating of the fruit of this tree is clearly a metaphor for the act of committing the first sin. As argued above, the first sin very likely leads to an inevitable fall from holiness because of the strong state-dependence of the ability to repent and the rate of continued sinning. In the Lutheran liturgy the following words, apparently adapted from those of Saint Ignatius of Loyola¹⁰ are quoted ritualistically:

Most merciful God, we confess that we are in bondage to sin *and cannot free ourselves*. We have sinned against you in thought, word, and deed, by what we have done and by what we have left undone.¹¹

The idea that we are unable to extract ourselves from the consequences of our sins is a critical component of Christian theology.

The Role of Jesus Christ

“You see, at just the right time, when we were still powerless, Christ died for the ungodly.” (Romans 5:6)

¹⁰Saint Ignatius of Loyola (1491–1556) was the founder of the Society of Jesus (“Jesuits”).

¹¹“Brief Order of Confession and Forgiveness,” *Lutheran Book of Worship* (Minneapolis: Augsburg Fortress, 1978), emphasis added.

If it is impossible for Man to extract himself from his state of sin, or unholiness, as modeled in the equations presented above, how can the original holiness of Man, before the original sin, be recovered? The Christian answer is that God became man in the form of Jesus Christ, “to give His life as a ransom for many” (Mark 10:45).

God, by His nature, is holy. Scripture says that “all He does is in truth and uprightness” (Psalm 111:7). If Christianity is true, it is possible that there is only one way to restore the holiness of Man, and that is to have Someone “lift” the weight of our sin from us, by taking it upon Himself. This suggests the addition of a second equation to the model. In the spirit of the first, this second equation describes the holiness, or perfection, of Christ, which we denote by P_c . Initially, then, before the Fall of Man, we have

$$\frac{dP_c}{dt'} = 0, P_c = 1 \quad (9a)$$

$$\frac{dP_m}{dt'} = 0, P_m = 1 \quad (9b)$$

Figure 4 shows the Holiness of Man, P_m as the x-axis, and the Holiness of Christ, P_c , as the y-axis. Note that before the Fall of Man, both Christ and Man were holy (Point A).¹² Then at some point, possibly shortly after the evolution of consciousness in Man, the original sin was committed, which dropped P_m below the point of no return (Point B), leading to the inevitable Fall of Man, toward Point C. At this point the second of the equations (9) is modified as follows:

$$\frac{dP_c}{dt'} = 0, P_c = 1 \quad (10a)$$

$$\frac{dP_m}{dt'} = 2P_m^n - 1 \quad (10b)$$

These equations apply while Man falls from point B to point C.

Subsequently, according to the tenets of Christianity, a new dynamic enters, when Christ is crucified for the sins of humanity: “In the fullness of time God sent forth his Son” (Galatians 4:4). Isaiah 53:56 says, “God has laid on Him, the iniquity of us all.” Here is a great mystery. Christians would say that Christ was without sin during His entire earthly life, but clearly He suffered and died, in the physical sense, when He took upon Himself the sins of the world. Therefore, since sin, as mentioned above, is scripturally

¹²Christ existed before all things: “I tell you the truth,” Jesus answered, “before Abraham was born, I am!” (John 8:58).

related to death, which in some sense is synonymous with loss of holiness, it is not unreasonable to include a term in equation 10a describing Christ's taking of our sins upon Himself during His crucifixion. Quantitatively, the cumulative sins of Man are given by $(1-P_m)$. Written in the non-dimensional timescale t' , we then have:

$$\frac{dP_c}{dt'} = - \left(\frac{\tau}{\tau_{cruc}} \right) (1-P_m) \quad (10a)$$

$$\frac{dP_m}{dt'} = (2P_m - 1) + \left(\frac{\tau}{\tau_{cruc}} \right) (1-P_m) \quad (10b)$$

Equations 10a and 10b are coupled differential equations that apply during the time that Christ is being crucified. Because the model is described on the fabric of time, the curious ratio τ / τ_{cruc} appears in the equations. This is the ratio of the time over which a perfect being loses all holiness, while sinning at the maximal rate, to the time during which Christ was crucified. One might imagine that this is a large ratio, because Christ was crucified in a day, while we do not know how long it takes for a being to fall completely from holiness.¹³ While one should probably not place too much emphasis on the details of this simple model, it is interesting to note that if τ / τ_{cruc} is very small, the equations predict that the sacrifice of Christ would not help, because the second term in (10b) would remain small compared to the first. The implication is that crucifixion of Christ had to happen quickly, or it would not work!

The last component to this model regards the resurrection of Christ, which represents return to life or holiness. How does this happen? Again, C. S. Lewis, in one of his best-known works of fiction, suggests an answer:

“But what does it all mean?” asked Susan when they were somewhat calmer. “It means,” said Aslan, “that though the Witch knew the Deep Magic, there is a magic deeper still which she did not know. Her knowledge goes back only to the dawn of time. But if she could have looked a little further back, into the stillness and the darkness before Time dawned, she would have read there a different incantation. She would have known that when a willing victim who had committed no treachery was killed in a traitor's stead, the Table would crack and Death itself would start working backward.”¹⁴

¹³This is, incidentally, what Christian doctrine holds happened to Satan: “He replied, ‘I saw Satan fall like lightning from heaven’” (Luke 10:18).

¹⁴C. S. Lewis, *The Lion the Witch and the Wardrobe* (New York: The MacMillan Company, 1950).

The idea presented in this fictional story is consistent with Scripture: “Greater love has no man than to lay down his life for his friends” (John 15:13).

If Someone takes the burden of another upon Himself, our innate sense of morality tells us that the person who does this willingly, while he or she may in fact suffer and die, does not truly lose his or her integrity, or holiness, but rather something like the reverse must be true. I propose that the act of taking another’s sins may be a form of “negative death,” or in other words, a form of *increase in life* rather than death. In game theory, this amounts to a “non-zero sum game” where both “players” may win. This may sound implausible to a student of science, but keep in mind that we are dealing with the spiritual world here, most of the laws of which are unknown. Luke 17:33 states, “Whoever seeks to keep his life will lose it, and whoever loses his life will preserve it.”

So, the final modification to the equations would include a term in the dPc/dt equation that is the negative of that given in 10a. How this might be quantified is more extreme speculation than has been presented thus far, but it plausibly occurs with a time delay, where the benefit does not occur until physical death. Scripture says “Lay not up for yourselves treasure on Earth . . . but lay up for yourselves treasure in heaven.” If we are storing treasure in heaven, how can we benefit from this while still on earth, i.e., before we die, physically? Thus, as Christ takes the sin of the world upon Himself (just after point C in the trajectory shown in Figure 4) his holiness decreases to zero, then is restored by the “negative death” associated with the taking of another’s sin upon one’s self. After physical death, Christ’s resurrection represents a recovery of his original holiness (the second part of the C- back-to-A trajectory).

Since Christ was perfectly holy in the beginning, and He knew no sin (“He made Him who knew no sin to be sin on our behalf, so that we might become the righteousness of God in Him,” 2 Corinthians 5:21), the net effect of his sacrifice, or “negative death” would seem to quantitatively result in an ascent to something higher than His original holiness, which I have already defined as “perfect holiness.” Thus, this is impossible given the definition used here. But this model is of course a tremendous simplification of the full explanation; in other words, many relevant variables are missing. For example, although there may be no sin in heaven, Scripture does state that there will be levels of greatness in heaven (e.g., Matthew 5:19, “but whoever practices and teaches these commands will be called great in the kingdom of heaven”). By taking the sins of the entire world upon Himself, then, He would certainly attain the “name above all names” status: “Therefore God exalted him to the highest place and gave him the name that is above every name, that at the

name of Jesus every knee should bow, in heaven and on earth and under the earth” (Philippians 2:9-10).

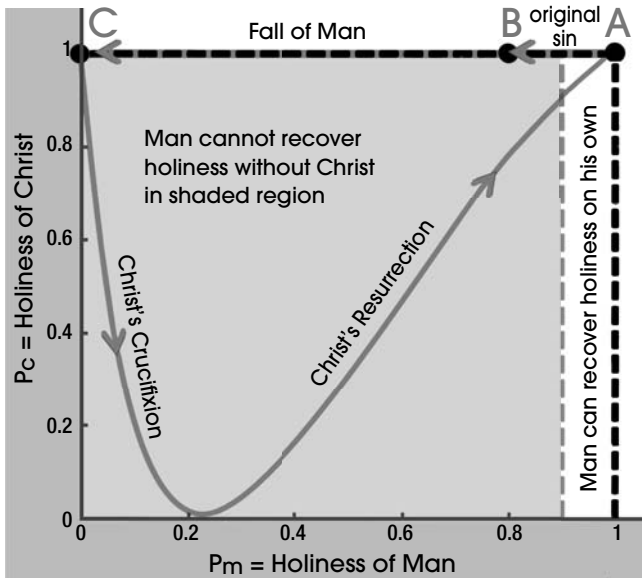


Figure 4: The Holiness of Man plotted versus the Holiness of Christ. See text.

The Result of Christ's Sacrifice on Man's Condition and Behavior

According to Christian doctrine, all that has to be done in order to have one's sins taken away by Christ is to accept and acknowledge Christ as one's own savior. The person is then said to be "in Christ." The state of being "in Christ" may be much like what is commonly called a "quantum state" in physics—a condition that can be occupied, not by an elementary particle, but instead by a conscious being. Notice the consequence of occupying this "spiritual state": The rate at which a conscious being sins, according to this model, is dependent on its state of holiness. Therefore, if one's sins have been "taken away" by Christ, it should be easier to avoid sin and live a holy life. Many have found this to be true, and therefore find great power in the redemption of Christ. Of course, one of the greatest criticisms against Christianity is the abundance of hypocrisy in the church. This is possible because one of the variables suppressed here is *volition* – the fact that every conscious being still has a free will, which allows one to choose to use the state of holiness achieved by being "in Christ" to do good, or, alternatively, to disregard the power to live righteously.

Conclusion

The model presented here attempts to shed light on the mysterious Christian core belief, wherein only the sacrifice of a perfect being (Christ) could atone for the Sin of the World. This model may seem to some to be self-contradictory in that a mathematical model, which is by nature precise in its predictions, is employed to describe the spiritual dynamics behind a concept for which rigorous quantification, by the author's own admission, is very likely impossible. But such is the nature of models in the physical sciences. They are generally useful only to a point, and their descriptive power usually lies in their simplicity—a simplicity that allows the mind of Man to grasp essential aspects of much deeper and more complex truths.

The Bohr model of the atom, for example, allowed some of our earliest insights into the inscrutable world of quantum mechanics. Eventually it had to be replaced by better models, but it has never lost its pedagogical value, which was part and parcel of its simplicity. This having been noted, there are numerous ways in which the present model could potentially be extended, with the possibility of additional descriptive power, and the concomitant risk of obfuscation of any value the model currently enjoys due to its simplicity. For example, perhaps one could extend this model by allowing a being to have a complex holiness variable: $P = P_{physical} + iP_{spiritual}$. This may allow a description of Christ deteriorating physically when crucified, while maintaining perfection spiritually. Furthermore, the idea that self-sacrifice leads to a “delayed reward” or “treasure in heaven” (cf. Matthew 6:19–21) is intriguing because in the complex plane, a time delay is the same thing as a phase shift. It is presently unclear whether pursuit of such extensions is worth considering.